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VRONSKIY DETERMINANTI VA UNING DIFFERENSIAL TENGLAMALAR NAZARIYASIDAGI ROLI

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Annotatsiya. *Ushbu maqolada Vronskiy determinanti va uning differensial tenglamalar nazariyasidagi roli tahlil qilingan. Vronskiy determinanti funksiyalar sistemasining chiziqli mustaqilligini aniqlashda asosiy vosita sifatida xizmat qiladi va yuqori tartibli chiziqli differensial tenglamalarning umumiy yechimini qurishda muhim ahamiyatga ega.*

Maqolada Wronskiy determinantining ta'rifi, asosiy xususiyatlari va chiziqli mustaqillikni tekshirishdagi ahamiyati keltirilgan. Shuningdek, ikki va uch funksiyali misollar yordamida determinantni hisoblash jarayoni bosqichma-bosqich ko'rsatilib, nazariy tushunchaning amaliy qo'llanilishi tasvirlanadi.

Kalit so'zlar: *Vronskiy determinanti, chiziqli mustaqillik, differensial tenglamalar, fundamental yechimlar, yuqori tartibli tenglamalar.*

Аннотация. *В данной статье анализируется детерминант Вронского и его роль в теории дифференциальных уравнений. Детерминант Вронского служит основным инструментом для определения линейной независимости системы функций и играет важную роль при построении общего решения линейных дифференциальных уравнений высокого порядка.*

В статье приведено определение детерминанта Вронского, его основные свойства и значение для проверки линейной независимости. Кроме того, с помощью примеров с двумя и тремя функциями пошагово демонстрируется процесс вычисления детерминанта и практическое применение теоретической концепции.

Ключевые слова: *детерминант Вронского, линейная независимость, дифференциальные уравнения, фундаментальные решения, дифференциальные уравнения высокого порядка.*

Abstract. *This article examines the Wronskian determinant and its role in the theory of differential equations. The Wronskian determinant serves as a fundamental tool for determining the linear independence of a system of functions and plays an important role in constructing the general solution of higher-order linear differential equations.*

The article presents the definition of the Wronskian determinant, its main properties, and its significance for testing linear independence. Furthermore, through examples involving two and three functions, the step-by-step process of calculating the determinant is demonstrated, illustrating the practical application of the theoretical concept.

Keywords: *Wronskian determinant, linear independence, differential equations, fundamental solutions, higher-order differential equations.*

Differensial tenglamalar nazariyasi matematik fizika, muhandislik va boshqa ilmiy sohalarda fundamental ahamiyatga ega. Ayniqsa, yuqori tartibli chiziqli differensial tenglamalarning yechimlarini o‘rganishda, yechimlarning chiziqli mustaqilligi va ularning fundamental sistemasini aniqlash muhim hisoblanadi.

Shu maqsadda **Vronskiy determinanti** (keyingi o‘rinlarda $W(y_1, y_2, \dots, y_n)$ bilan belgilanadi) asosiy vosita sifatida ishlatiladi. Vronskiy determinanti funksiyalar sistemasining chiziqli mustaqilligini tekshirish va differensial tenglamaning umumiy yechimi strukturasi haqida xulosa chiqarish imkonini beradi.

Ta’rif: Agar $y_1(x), y_2(x), \dots, y_n(x)$ funksiyalar berilgan bo‘lsa, ularning Vronskiy determinanti quyidagicha aniqlanadi:

$$W(y_1, y_2, \dots, y_n)(x) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_1^{(n-1)} \end{vmatrix}$$

Asosiy xususiyatlar:

1. Agar $W(x_0) \neq 0$ bo‘lsa, funksiyalar $y_1, y_2, \dots, y_n$ **chiziqli mustaqil** hisoblanadi.
2. Agar barcha nuqtalarda $W(x_0) = 0$ bo‘lsa, funksiyalar chiziqli bog‘liq bo‘lishi mumkin (biroq bu har doim ham shart emas; qo‘shimcha shartlar talab qilinadi).
3. Wronskiy determinanti yordamida differensial tenglamaning fundamental yechimlar sistemasini aniqlash mumkin.

Misol. Vronskiy determinantini tuzing.

$y_1 = e^x, \qquad y_2 = xe^x, \qquad y_3 = x^2e^x$

1-qadam. Hosilalarni topamiz.

$y_1' = e^x, \qquad y_1'' = e^x$

$y_2' = (1+x)e^x, \qquad y_2'' = (2+x)e^x$

$y_3' = (2x+x^2)e^x, \qquad y_3'' = (2+4x+x^2)e^x$

2-qadam. Wronskiy determinantini tuzamiz.

$$W = \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & (1+x)e^x & (2x+x^2)e^x \\ e^x & (2+x)e^x & (2+4x+x^2)e^x \end{vmatrix}$$

3-qadam. Umumiy ko‘paytuvchini ajratamiz

$$W = e^x \begin{vmatrix} 1 & x & x^2 \\ 1 & 1+x & 2x+x^2 \\ 1 & 2+x & 2+4x+x^2 \end{vmatrix}$$

4-qadam. Soddalashtiramiz va hisoblaymiz.

2- va 3-satrlardan 1-satrni ayiramiz.

$$W = e^x \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 2 & 2+4x \end{vmatrix} = e^x \begin{vmatrix} 1 & 2x \\ 2 & 2+4x \end{vmatrix} = e^x(2+4x-4x) = 2e^x$$

$W \neq 0 \rightarrow \mathit{funksiyalar\ chiziqli\ mustaqil}$

Xulosa:

$y_1 = e^x, \qquad y_2 = xe^x, \qquad y_3 = x^2e^x$

$W \neq 0 \rightarrow funksiyalar\ chiziqli\ mustaqil$

— quyidagi differensial tenglamaning fundamental yechimlari:

$y''' - 3y'' + 3y' - y = 0$

Vronskiy determinanti asosan differensial tenglamalar va funksiyalar nazariyasida ishlatiladi. Uning asosiy maqsadlari quyidagilar:

1. Chiziqli mustaqillikni tekshirish
2. Differensial tenglamalarda fundamental yechimlarni toppish
3. Yechimlar strukturasi va tartibini aniqlash
4. Differensial tenglamalarni yechishda yordamchi vosita

Vronskiy determinanti asosan **matematika va amaliy fanlarda**, ayniqsa funksiyalar va differensial tenglamalar bilan bog‘liq sohalarda qo‘llaniladi.

1. Differensial tenglamalar nazariyasi
2. Matematik fizika
3. Chiziqli algebra va funksional analiz
4. Nazariy fizika
5. Muhandislik fanlari
6. Amaliy matematika va hisoblash matematikasi

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