METHODOLOGICAL FOUNDATIONS FOR EXPLAINING THE CONCEPT OF FUNCTION TO STUDENTS

Boltaboyev Ixtiyorjon Baxtiyorovich
Mathematics Teacher, Department of Exact Sciences, Academic Lyceum of Fergana State University, Uzbekistan.
https://doi.org/10.5281/zenodo.20538525

Abstract. *This article discusses the theoretical, didactic, and methodological foundations for explaining the concept of function to students in mathematics education. The concept of function is one of the fundamental mathematical concepts directly related to algebra, geometry, mathematical analysis, physics, computer science, economics, and technical sciences. One of the important tasks of mathematics education is to ensure that students understand this concept not superficially, merely as a formula or a graph, but as a relationship between quantities, a correspondence, a process of change, and a means of modeling. The article analyzes the use of real-life situations in forming the concept of function, the establishment of connections among multiple representations such as tables, graphs, formulas, and verbal descriptions, the development of students' conceptual understanding, the early identification and correction of errors, and the application of information and communication technologies as well as dynamic mathematical software. In addition, the importance of such principles as gradualness, visualization, problem-based learning, practical orientation, interdisciplinary integration, and a competency-based approach in teaching the concept of function is substantiated. The article also presents a methodological model for organizing a lesson, typical difficulties encountered by students, and ways to overcome them.*

Keywords: *function, concept of function, methodology of teaching mathematics, algebra, graph, table, formula, domain, range, mathematical modeling, multi-representational approach, critical thinking, GeoGebra, competency-based approach.*

Introduction

In mathematics education, the concept of function is one of the most important, most widely used, and foundational concepts that connects many other mathematical topics. As students study such topics in algebra as equations, inequalities, expressions, polynomials, rational expressions, and progressions, they gradually move toward understanding relationships between quantities. Among these relationships, the concept of function occupies a special place.

This is because a function expresses how one quantity changes depending on another quantity; it makes it possible to transform real-life processes into mathematical models, observe changes through graphs and tables, perform calculations through formulas, and explain the meaning of a phenomenon through verbal descriptions.

The main problem in explaining the concept of function to students is that many students perceive a function only as a formula of the form "y = ...". For some students, a function is merely a line drawn on the coordinate plane, while for others it remains only a set of x and y values in a table. Such a superficial understanding creates serious difficulties in later grades when students study linear, quadratic, power, exponential, logarithmic, and trigonometric functions, especially when they learn such concepts as derivative, integral, limit, extremum, monotonicity, and continuity. Therefore, in teaching the concept of function, a teacher should not be limited to simply stating the definition; rather, the teacher must form in the student's mind a complete and stable conceptual image of this concept.

The concept of function also plays an important role in developing students' mathematical thinking. Through this concept, students learn to answer such questions as: "If one quantity changes, how does the other change?", "How can a relationship be expressed?", "What does a graph show?", "What conclusion can be drawn from a table?", and "What does each symbol in a formula represent?" Thus, the topic of function teaches students not only to calculate, but also to analyze, compare, model, make assumptions, and draw conclusions.

The relevance of this article is also reflected precisely in this aspect: modern mathematics education requires students not merely to memorize ready-made formulas, but to understand mathematical concepts and apply them in real-life situations. In the education system of Uzbekistan, state educational standards and mathematics curricula also pay considerable attention to the development of students' logical thinking, independent work, mathematical literacy, and ability to solve practical problems. From this point of view, explaining the concept of function to students in a methodologically sound way plays an important role in improving the quality of mathematics lessons, developing students' mathematical literacy, and preparing them for more complex topics in the future.

Main Part

The mathematical and didactic essence of the concept of function

From a mathematical point of view, a function is a relation that assigns to each element of one set exactly one element of another set. In the school algebra course, this concept is often explained in simpler language: "If each value of x corresponds to exactly one value of y, then y is called a function of x." This definition is convenient for students, but merely stating it verbally is not sufficient. The teacher must explain each component of the definition separately: what is meant by "a value of x," what "correspondence" means, why the condition of "one value" is important, what happens if two different y values correspond to one x value, and what the difference is between a function and an ordinary relation. All of these aspects must be clearly formed in students' minds.

The didactic essence of the concept of function lies in the fact that it develops students' understanding of variable quantities. For example, the distance traveled by a car depends on time, the cost of purchased goods depends on their quantity, air temperature changes over time during the day, the temperature changes over time during the boiling of water, and payment in a phone tariff depends on the number of minutes used. All these real-life situations lead to the concept of function. At first, the student observes a real situation, then constructs a table, then draws a graph, after that writes a formula, and finally makes a general conclusion. Such a sequence helps the concept of function develop naturally.

An important methodological requirement in teaching the topic of function is that the teacher should not begin immediately with an abstract definition. An abstract definition is a ready-made mathematical result, and in order for students to accept it consciously, they must first pass through the stages of experience, observation, examples, relationships, and generalization. For instance, the teacher may first present the following situation: "One notebook costs 3,000 soums.

ISSN: 2181-3906 2026

International scientific journal «MODERN SCIENCE AND RESEARCH»
VOLUME 5 / ISSUE 6 / UIF:8.2 / MODERNSCIENCE.UZ

What will be the total cost if 1, 2, 3, or 4 notebooks are purchased?" Students construct a table: the number of notebooks is 1, 2, 3, 4, while the cost is 3,000, 6,000, 9,000, 12,000. Then they conclude that "when the number of notebooks changes, the total cost also changes." Next, this relationship is expressed by the formula $y = 3000x$. Only after this should students be led to the idea of a function by explaining that "each value of x corresponds to exactly one value of y."

Stages of forming the concept of function

The explanation of the concept of function to students should be organized step by step.

The first stage is the observation of real-life relationships. At this stage, students observe relationships between two quantities in the phenomena around them. For example, relationships such as time and distance, number of products and price, working time and amount of work completed, air temperature and time, side length and area of a square, radius and circumference of a circle can be examined. The main purpose of this stage is to instill in students the idea that "a change in one quantity may lead to a change in another quantity."

The second stage is representation through a table. Students organize numerical data from a real-life situation. A table is a very convenient tool for forming the concept of function because it makes the correspondence between x and y values visible. When working with a table, the teacher asks students the following questions: How are the x values changing? How are the y values changing? How many y values correspond to each x value? Can two different y values correspond to one x value? Is there a pattern in the table? These questions encourage students not merely to calculate, but to understand the relationship.

The third stage is representation through a graph. The graph of a function allows students to see the relationship visually. However, when teaching graphs, the teacher should not be limited to marking points and connecting them. Students must understand the meaning of the graph: what the horizontal axis represents, what the vertical axis represents, what each point on the graph expresses, what it means when the graph rises, what it means when it falls, and what real-life situations correspond to a straight line, a curve, or a piecewise graph. These questions develop students' graphical literacy.

The fourth stage is generalization through a formula. A formula is a concise and general form of a functional relationship. For example, the formula $y = 2x + 3$ makes it possible to find y for each value of x. However, students must understand that a formula is not only a tool for calculation, but also a model of a relationship. What the coefficients in the formula represent, how y changes when x changes, whether a table can be constructed using the formula, whether a graph can be drawn from the formula, and whether it is possible to move from a graph to a formula are questions that strengthen the connection between the formula and its meaning.

The fifth stage is definition and generalization. It is advisable to give the general definition only after students have worked with the above-mentioned experience, tables, graphs, and formulas. At this stage, the teacher explains the definition of a function not formally, but meaningfully: "If each value of x corresponds to exactly one value of y, then y is a function of x."

Then such concepts as function, argument, value of the function, domain, and range are introduced gradually. If too many of these concepts are introduced in one lesson, students may memorize them mechanically. Therefore, each new term should be reinforced with examples and practical tasks.

Teaching the concept of function through a multi-representational approach

One of the most effective methodological approaches in mastering the concept of function is the multi-representational approach. According to this approach, a function is studied simultaneously through a verbal description, table, graph, formula, correspondence scheme, and real-life situation. If a student can see one relationship in different forms, his or her understanding of the function becomes deeper. For example, the verbal situation “each kilogram of apples costs 12,000 soums” is transformed into a table, the formula $y = 12000x$, and a straight line on the coordinate plane. In this process, the student understands that the same mathematical meaning can be expressed in different forms.

The main advantage of the multi-representational approach is that it develops not only students’ algebraic thinking, but also their visual, logical, and practical thinking. Some students understand a formula quickly but struggle to read a graph. Other students understand a graph well but make mistakes when constructing a formula. Still others can find a pattern in a table but cannot write it in the form of a general formula. Therefore, the teacher should systematically organize exercises that require movement among all representations: from a word problem to a table, from a table to a graph, from a graph to a formula, from a formula to a table, and from a graph to an oral explanation. Such exercises strengthen students’ understanding of the concept of function.

This methodology is especially effective when teaching the topic of linear functions. For example, when the function $y = 2x + 1$ is given, students first choose several values of x and construct a table; then they place the points on the coordinate plane, form the graph, and after that draw conclusions about the slope of the graph and the point where it intersects the y-axis.

Then it is explained that the number “2” means that when x increases by one unit, y increases by two units, while the number “1” indicates where the graph intersects the y-axis. In this case, the formula does not remain a mechanical expression; it acquires meaning.

Typical difficulties encountered by students

When learning the concept of function, students often make several typical mistakes. The first mistake is confusing a function with an equation. When a student sees $y = 2x + 3$, he or she may sometimes treat it as an ordinary equation and try to find x. In such a situation, the teacher should explain that in a function formula, x is the independent variable and y is the dependent variable. This formula does not serve to find one unknown; rather, it serves to determine the corresponding values of y for different values of x.

The second mistake is assuming that any graph is a function. A student may consider any line or curve drawn on the coordinate plane to be the graph of a function. To prevent this, the meaning of the “vertical line test” can be explained in a simple way: if two y values on the graph correspond to one x value, then it is not a function. For example, in the graph of a circle, some x values correspond to two different y values; therefore, a circle is not considered a single-valued function of y with respect to x. Such examples reinforce the “unique value” condition in the definition of a function.

The third mistake is failing to distinguish between the domain and the range. Students often confuse the values that x can take with the values that y can produce. To overcome this, the teacher should use real-life situations.

For example, the “number of students in a classroom” cannot be negative or fractional; therefore, in a practical situation, the values of x are limited. Or, if the area of a square is expressed by the formula $y = x^2$, then x cannot be negative because it represents length. Such examples show that the domain is related not only to algebraic conditions, but also to meaningful conditions.

The fourth mistake is seeing a graph only as a drawing and being unable to read its meaning. Many students can draw a graph, but they have difficulty obtaining information from it.

For example, understanding where the graph is increasing, where it is decreasing, at which point the value is greater, in which interval the change is faster, and what the graph represents in a real-life situation requires graphical literacy. Therefore, the teacher should regularly use not only graph-drawing exercises, but also graph-interpretation exercises.

Using problem-based learning in teaching the concept of function

Problem-based learning is one of the most effective methods in teaching the concept of function. In this approach, the teacher does not give students the ready-made definition, but leads them to the definition through a problem situation. For example, at the beginning of the lesson, the following task is given: “In a taxi service, the initial fee is 8,000 soums, and 3,000 soums are charged for each kilometer. If the distance is x kilometers, how can the total payment be determined?” Students first calculate the payment for several values, then construct a table, and finally arrive at the formula $y = 8000 + 3000x$. After that, the teacher asks: “In this situation, how many payment values correspond to each value of x?” Students conclude that there is “one” value.

This conclusion leads them to the definition of a function.

The advantage of problem-based learning is that it creates a need for knowledge in students. Students feel that the definition of function is needed not for memorization, but for solving a real problem. This increases their interest in the topic. In addition, problem-based learning teaches students to ask questions, make assumptions, discuss, and justify their opinions with evidence. In the topic of function, questions such as “Is every relationship a function?”, “What happens if two y values correspond to one x value?”, “Can there be a function without a graph?”, and “Can a function be given without a formula?” make the lesson more active and deepen the concept.

Opportunities of information and communication technologies and GeoGebra

In modern mathematics lessons, the use of information and communication technologies, especially dynamic mathematical software such as GeoGebra, is highly effective in explaining the concept of function. With the help of GeoGebra, students can quickly draw the graph of a function, change parameters, and observe shifts of the graph, slope, intersection points, maximum and minimum points, and other properties. Such a visual and dynamic approach is especially convenient in explaining linear, quadratic, exponential, and trigonometric functions.

For example, when studying the function $y = ax + b$, sliders can be created in GeoGebra for the parameters a and b. Students observe that when the value of a changes, the slope of the graph changes, and when the value of b changes, the graph shifts upward or downward. This process strengthens students’ understanding that “coefficients affect the shape and position of the graph.” Similarly, in the function $y = ax^2$, the positive or negative value of a shows the direction of the parabola, while the absolute value of a shows whether it is wider or narrower.

This can be shown on an ordinary blackboard, but through a dynamic program students observe the change directly.

However, there is an important methodological condition in using ICT tools: the program should be a tool, not the goal. If students only see a ready-made graph but do not understand its mathematical meaning, technology will not produce the expected result. Therefore, when using GeoGebra or any other program, the teacher must connect each observation with a question: "What changed?", "Why did it change?", "Which parameter caused it?", "How can this be seen from the formula?", and "What does this graph represent in a real-life situation?" Only then will technology develop students' conceptual understanding.

Teaching the concept of function through interdisciplinary integration

The concept of function is not only a topic in algebra; it is also a concept of interdisciplinary significance. In physics, the dependence of distance on time, the change of velocity over time, and the dependence of electric current on voltage; in biology, the dependence of an organism's growth on time; in geography, the change of temperature across seasons; in economics, demand and supply; and in computer science, the dependence of an algorithm's output on the input value are all explained through functional relationships. Therefore, teaching the topic of function through interdisciplinary integration helps students understand the topic practically.

For example, in integration with physics, the model of uniform motion is explained through the formula $s = vt$. Here, time is taken as x, and the distance traveled is taken as y. If velocity is constant, then distance depends linearly on time. In biology, the height of a plant changing over time is represented through a table and a graph. In an example related to economics, functional thinking is formed through such concepts as the relationship between product price and quantity, discount, income, and expenses. Such an integrative approach shows students that the concept of function is not an abstract idea detached from life, but a means of understanding real processes.

A methodological model for organizing the lesson

The following methodological model can be proposed for teaching the concept of function.

The first stage of the lesson is a motivational introduction, in which the teacher presents a real-life situation. For example, a case such as "In an internet tariff, the monthly subscription fee is 20,000 soums, and 5,000 soums are paid for each additional gigabyte. How can the total cost be found?" arouses students' interest.

At the second stage, students construct a table. They calculate the total cost for 0, 1, 2, 3, and 4 gigabytes. Through the table, the correspondence between x and y becomes visible. At the third stage, a graph is drawn based on this table. The graph shows students the direction of change.

At the fourth stage, a formula is constructed: $y = 20000 + 5000x$. At the fifth stage, the definition of a function is given and the main terms are introduced. At the sixth stage, reinforcement exercises are completed: determining whether a given table represents a function, whether a graph represents a function, finding values using a formula, reading values from a graph, and constructing a formula suitable for a real-life situation.

At the seventh stage, reflection is organized. Students are asked questions such as: "What did you learn about functions today?", "In what ways can a function be represented?", "Why is it not a function if two y values correspond to one x value?", and "What does a graph show?" This stage helps determine the extent to which students have understood the topic.

Assessment criteria

In assessing students' mastery of the concept of function, it is not enough to rely only on calculation tasks. Assessment should be multi-criteria. The first criterion is whether the student can explain the definition of a function. The second criterion is whether the student can move among a table, graph, formula, and verbal description. The third criterion is whether the student can determine whether a given relationship is or is not a function and explain the reason. The fourth criterion is whether the student can distinguish between the domain and the range. The fifth criterion is whether the student can construct a function model suitable for a real-life situation.

The sixth criterion is whether the student can not only draw a graph, but also interpret it meaningfully.

Such assessment determines not only students' mechanical knowledge, but also their conceptual understanding, logical thinking, and practical application skills. For example, the task "Find y if x = 4 in the function y = 3x + 2" is necessary, but it is not sufficient. In addition, students should be asked questions such as: "What real-life situation may this formula correspond to?", "What will the graph look like?", "How does y change when x increases by one unit?", and "What values of x are meaningful in this situation?"

Conclusion

Explaining the concept of function to students is one of the most important methodological issues in mathematics education. This is because a function is not only a separate topic in the algebra course, but also one of the main connecting concepts of the whole field of mathematics. If students deeply understand the concept of function, they will later master linear, quadratic, power, exponential, logarithmic, and trigonometric functions, as well as such complex topics as limits, derivatives, integrals, and mathematical modeling more easily.

As analyzed in the article, in order to teach the concept of function effectively, it is necessary to begin with real-life situations and then establish connections among tables, graphs, formulas, and verbal descriptions. Instead of giving the definition of a function in ready-made form at the beginning of the lesson, it is more appropriate to involve students in the process of observing real relationships, constructing tables, drawing graphs, forming formulas, and making general conclusions. Such an approach contributes to students' conscious mastery of the topic.

In explaining the concept of function, the multi-representational approach, problem-based learning, interdisciplinary integration, the development of graphical literacy, the use of dynamic mathematical software such as GeoGebra, and the early identification and elimination of typical mistakes are of great methodological importance. During the lesson, the teacher should teach students not only to calculate, but also to see, explain, model, and analyze relationships. Only then will the concept of function be formed in students' minds not as a simple formula, but as a means of mathematically expressing changes in the real world.

In general, the methodology of teaching the concept of function serves to develop students' logical, functional, graphical, and modeling thinking. This is directly connected with the main goals of modern mathematics education: forming mathematical literacy, independent thinking, the ability to analyze real-life situations, and competencies for applying knowledge in practice.

References

1. Harel G., Dubinsky E. eds. The Concept of Function: Aspects of Epistemology and Pedagogy. MAA Notes, Vol. 25. – Washington, DC: Mathematical Association of America, 1992.
2. Polya G. How to Solve It: A New Aspect of Mathematical Method. – Princeton: Princeton University Press, 1945.
3. Skemp R.R. The Psychology of Learning Mathematics. – Harmondsworth: Penguin Books, 1971.
4. Schoenfeld A.H. Mathematical Problem Solving. – Orlando: Academic Press, 1985.
5. NCTM. Principles and Standards for School Mathematics. – Reston, VA: National Council of Teachers of Mathematics, 2000.
6. Stewart J. Calculus: Early Transcendentals. – Belmont: Brooks/Cole, 2012.
7. Gaziyev A., Israilov I., Yaxshiboyev M. Functions and Graphs: A Textbook for Higher Educational Institutions. – Tashkent: Voris Publishing House, 2006.
8. Akmalov A. et al. Algebra. Grade 7: Textbook for the 7th Grade of General Secondary Schools. – Tashkent: Republican Education Center, 2022.
9. Mirzaahmedov M.A., Ismailov Sh.N., Amanov A.Q. Mathematics. Grade 10. Algebra and Fundamentals of Analysis. Geometry. Part I. – Tashkent, 2017.
10. State Educational Standards of General Secondary and Secondary Specialized Education approved by Resolution No. 187 of the Cabinet of Ministers of the Republic of Uzbekistan dated April 6, 2017.
11. II. Scientific Articles
12. Tall D., Vinner S. Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity // Educational Studies in Mathematics. – 1981. – Vol. 12, No. 2. – P. 151–169. DOI: 10.1007/BF00305619.
13. Vinner S., Dreyfus T. Images and Definitions for the Concept of Function // Journal for Research in Mathematics Education. – 1989. – Vol. 20, No. 4. – P. 356–366.
14. Sfard A. On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin // Educational Studies in Mathematics. – 1991. – Vol. 22, No. 1. – P. 1–36. DOI: 10.1007/BF00302715.
15. Carlson M.P., Jacobs S., Coe E., Larsen S., Hsu E. Applying Covariational Reasoning While Modeling Dynamic Events: A Framework and a Study // Journal for Research in Mathematics Education. – 2002. – Vol. 33, No. 5. – P. 352–378.
16. Even R., Tirosh D. Subject-Matter Knowledge and Knowledge about Students as Sources of Teacher Presentations of the Subject-Matter // Educational Studies in Mathematics. – 1995. – Vol. 29. – P. 1–20. DOI: 10.1007/BF01273897.
17. Markovits Z., Eylon B., Bruckheimer M. Functions Today and Yesterday // For the Learning of Mathematics. – 1986. – Vol. 6, No. 2. – P. 18–24.
18. Lesh R., Post T., Behr M. Representations and Translations among Representations in Mathematics Learning and Problem Solving // In: Janvier C. ed. Problems of Representation in the Teaching and Learning of Mathematics. – Hillsdale, NJ: Lawrence Erlbaum, 1987. – P. 33–40.

19. Adu-Gyamfi K., Stiff L.V., Bossé M.J. Lost in Translation: Examining Translation Errors Associated with Mathematical Representations // School Science and Mathematics. – 2012. – Vol. 112, Issue 3. – P. 159–170.
20. Duval R. Registers of Semiotic Representations and Analysis of the Cognitive Functioning of Mathematical Thinking // In: Duval R. Understanding the Mathematical Way of Thinking: The Registers of Semiotic Representations. – Cham: Springer, 2017.
21. Reisman A. Reading Like a Historian: A Document-Based History Curriculum Intervention in Urban High Schools // Cognition and Instruction. – 2012. – Vol. 30, No. 1. – P. 86–112. DOI: 10.1080/07370008.2011.634081.
22. III. Internet Sites
23. Lex.uz
24. UZBMB
25. ZiyoNET
26. GeoGebra