QUADRATIC EQUATIONS: THEORY, DISCRIMINANT AND SOLUTION METHODS

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Abstract. *This article presents the fundamental concepts of quadratic equations, including their standard form, discriminant, and methods of finding solutions using the quadratic formula.*

The discriminant determines the nature and number of roots, categorizing them as distinct real, repeated, or complex. Step-by-step examples are provided to illustrate how the quadratic formula is applied. The aim is to give a clear and accessible explanation suitable for students and researchers studying algebra.

Keywords: *quadratic equation, discriminant, roots, quadratic formula, algebra.*

Mathematics is one of the oldest and most fundamental sciences in human history. It plays an essential role in many aspects of everyday life and in all fields of science and technology. From simple counting to advanced theories of the universe, mathematics helps us understand, describe, and predict natural phenomena.

At its core, mathematics is the study of numbers, shapes, patterns, and relationships. The subject can be divided into several branches, including arithmetic, algebra, geometry, trigonometry, calculus, and statistics. Each of these areas focuses on different mathematical concepts and tools that help solve various problems.

Arithmetic is the most basic branch of mathematics. It deals with numbers and the four basic operations: addition, subtraction, multiplication, and division. These skills are used daily, from shopping and cooking to managing time and money.

Algebra introduces letters and symbols to represent numbers and quantities in formulas and equations. This branch helps develop critical thinking and problem-solving skills. Geometry, on the other hand, focuses on shapes, sizes, and the properties of space. It is essential for fields such as architecture, engineering, and computer graphics.

Calculus is another important branch, developed to study change and motion. It is widely used in physics, engineering, economics, and even in predicting trends in business and medicine.

Statistics and probability help us analyze data, make predictions, and understand the likelihood of events. In our data-driven world, these tools are increasingly valuable.

Mathematics is not only about numbers; it is a way of thinking. It encourages logical reasoning, precision, and creativity. Students who study mathematics learn how to break complex problems into smaller, manageable steps. These skills are useful not only in academic pursuits but also in everyday decision-making.

In addition, mathematics provides the language for science and technology. Without it, modern inventions such as computers, smartphones, and satellites would not exist. Scientific discoveries in physics, chemistry, and biology also rely heavily on mathematical models and calculations.

In conclusion, mathematics is a universal language that connects all aspects of life and knowledge. Whether we realize it or not, we use mathematics every day.

By studying and appreciating this subject, we gain powerful tools to understand our world better and to solve the challenges of the future.

Introduction

In mathematics, a quadratic equation is an equation where the highest power of the unknown variable is two. This means the equation includes a squared term like x^2 Quadratic equations are widely used in algebra, physics, engineering, economics, and many real-world problems.

Standard Form

A quadratic equation is usually written in the following standard form:

$ax^2 + bx + c = 0$

Here:

x is the unknown variable,

a, b, and c are known numbers (coefficients), and $a \neq 0$

How Many Solutions?

Quadratic equations can have:

Two real solutions,

One real solution (called a double root), or

The discriminant is defined as:

$D = b^2 - 4ac$

Two complex solutions. The number and type of solutions depend on the discriminant:

If $D > 0$ the equation has two distinct real rioya.

If $D < 0$ the equation has two complex roots.

If there is one real root. $D = 0$

Quadratic Formula

The solutions (roots) of a quadratic equation can be found using the quadratic formula:

$x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula gives you the values of x that make the equation true.

Examples

Example 1 - Simple Quadratic

Solve:

$x^2 - 3x + 2 = 0$

Using the quadratic formula: Here a = 1 , b = - 3 c = 2

$x_{1,2} = \dfrac{3 \pm \sqrt{3^2 - 4*2*1}}{2*1}$

$x_1 = \dfrac{3+1}{2} = 2$

$x_2 = \dfrac{3-1}{2} = 1$

So the roots are:

and $x_1 = 2$ $x_2 = 1$ These are the solutions of the equation.

Example 2 - With Discriminant Zero

Solve:

$x^2 - 4x + 4 = 0$ Here, a = 1 , b = - 4 c = 4 The discriminant is:

D = (- 4)^2 - 4 * 1 * 4 = 16 - 16 = 0

Since D = 0 there is one real root:

x = 4/2 = 2

So the equation has one double root at x = 2

Conclusion

This article reviewed the basic principles of quadratic equations, including how the discriminant determines the nature of solutions and how the quadratic formula provides a universal method for solving any quadratic equation. These concepts are foundational in algebra and have important applications in various fields of science and engineering.

References

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