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BETA FUNKTSIYASINING O'ZIGA XOS USULLARI

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Beta Funktsiyasi.

Beta funksiyasi, ehtimollik nazariyasi va statistikada muhim ahamiyatga ega. U beta taqsimotini ifodalashda qo‘llaniladi, bu taqsimot esa o‘zgaruvchilarning chegarali intervaldagi taqsimlanishiga asoslangan modellarni yaratishda qo‘llanadi. Statistikada va genetika, molekulyar biologiya kabi sohalarda beta funksiyasi orqali optimallashtirish va parametrlarni baholash amalga oshiriladi.

Beta funksiyasi esa ikkita mustaqil o‘zgaruvchiga ega bo‘lib, quyidagicha ifodalanadi:

$$\boldsymbol{B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt,}\quad \textbf{x>0, y>0} \qquad \textbf{(5)}$$

xosmas integralga Eylerning Beta funksiyasi deyiladi. va nuqtalarga maxsus nuqtalar deyiladi.

(5) integralni ikkita integralning yig’indisi shaklida ifodalaymiz:

$$B(x,y) = \int_0^{\frac{1}{2}} t^{x-1}(1-t)^{y-1}dt + \int_{\frac{1}{2}}^1 t^{x-1}(1-t)^{y-1}dt,$$

bu yerda, Beta funksiya va da aniqlangan bo’lgani uchun, birinchi integral da, ikkinchi integral esa da yaqinlashuvchi bo’ladi.[1-10]

Beta funksiya xossalari.

1^0. B(x,y) = B(y,x); Haqiqatan $\tau = 1 - t$ deb o’zgaruvchini almashtirsak ,

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \int_0^1 \tau^{x-1}(1-\tau)^{y-1}dt = B(x,y).$$

2^0. Bo’laklab integrallash usuli yordamida (5) formuladan, $y > 1$bo’lganda

$t^x = t^{x-1} - t^{x-1}(1-t)$ ayniyatni qo’llab, quyidagini hosil qilamiz:

$$B(x,y) = \int_0^1 (1-t)^{y-1}d\left(\frac{t^x}{x}\right) = \frac{t^x(1-t)^{y-1}}{x}\Big|_0^1 + \frac{y-1}{x}\int_0^1 t^x(1-t)^{y-1}dt =$$

$$= \frac{y-1}{x}\int_0^1 t^x(1-t)^{y-1}dt - \frac{y-1}{x}\int_0^1 t^x(1-t)^{y-2}dt =$$

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$$= \frac{y-1}{x}B(x,y-1) - \frac{y-1}{x}B(x,y).$$

Bundan

$$\boldsymbol{B(x,y) = \frac{y-1}{x+y-1}B(x,y-1),\ x>0, y>1.}\quad \textbf{(6)}$$

Bu formulani $y > 1$ bo'lganda uni kichiklashtirish maqsadida qo'llash mumkin; shunday qilib hamma vaqt ikkinchi argument 1 dan kichik yoki teng bo'lishiga erishish mumkin. Xuddi shunday birinchi argumentga nisbatan ham mulohaza yuritib, $x > 1$ da

$$B(x,y) = \frac{x-1}{y+x-1}B(x-1,y), x>1, y>0. \quad (6')$$

(6) ga o'xshash keltirish formulasini olish mumkin. Agar natural son bo'lsa, (6) formulani qo'llab,

$$B(x,n) = \frac{n-1}{x+n-1}\cdot\frac{n-2}{x+n-2}\cdots\frac{1}{x+1}B(x,1)$$

$B(x,1) = \int_0^1 t^{x-1}dt = \frac{1}{x}$ ga ega bo'lamiz. Shuning uchun B(x,n) uchun ham B(n,x) uchun ham bir vaqtning o'zida

$$\boldsymbol{B(n,x) = B(x,n) = \frac{1\cdot 2\cdot 3\cdot \ldots\cdot (n-1)}{x\cdot(x+1)\cdot(x+2)\ldots\cdot(x+n-1)}} \quad \textbf{(7)}$$

ifodaga ega bo'lamiz.

Agar (7) da natural son m ga teng bo'lsa, u holda

$$B(m,n) = \frac{(n-1)!(m-1)!}{(m+n-1)!}$$

bo'ladi**.** [11-15]

3^0. Agar (6) integralda $t = \frac{u}{u+1}$ almashtirib olsak, bu yerda u– yangi o'zgaruvchi 0 dan $+\infty$ gacha o'zgaradi, u holda quyidagi formula o'rinli:

$$\boldsymbol{B(x,y) = \int_0^{+\infty}\frac{u^{x-1}}{(1+u)^{x+y}}du.} \quad \textbf{(8)}$$

4^0. (6) formulada $0 < x < 1$ bo'lganda, y=1-x deb olinsa,

$$\frac{1}{u+1} = \sum_{k=0}^{n}(-1)^k u^k + (-1)^{n+1}\frac{u^{n+1}}{1+u}du = \int_0^1 \frac{u^{x+1}+u^x}{1+u}du =$$

$$= (-1)^{n+1}\int_0^1 \frac{u}{u+1}(u^{x-1}+u^x)du + \sum_{k=0}^{n}(-1)^k u^k +$$

$+ (-1)^{n+1}\int_0^1 \frac{u^{n+1}}{u+1}$ ayniyatdan foydalanib,

$$B(x,1-x)=\int_0^{+\infty}\frac{u^{x-1}}{1+u}du$$

$$=\int_0^1\frac{u^{x-1}+u^{-x}}{1+u}du=(-1)^{n+1}\int_0^1\frac{u}{1+u}(u^{x-1}+u^{-x})du+$$

$+\sum_{k=0}^{n}(-1)^k(\frac{1}{k+x}+\frac{1}{k-x+1})$ (9)

ni olamiz:

$$0\le\int_0^1\frac{u^{n+1}(u^{x-1}+u^{-x})}{1+u}d\le\int_0^1(u^{n+x}+u^{n+1-x})du=$$

$$=\frac{1}{n+x+1}+\frac{1}{n+2-x}$$

bo‘lgani uchun (9) formulada $n\to\infty$ deb limitga o‘tilsa,

$$B(x,1)=\sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{x+k}+\frac{1}{k-x+1}\right)$$

$$=\frac{1}{x}+\sum_{k=0}^{\infty}(-1)^k\left(\frac{1}{x+k}+\frac{1}{x-k}\right)=\frac{\pi}{\sin\pi x}$$

ni hosil qilamiz. Oxirgi formula $\frac{1}{\sin z}$ funksiyaning elementlar kasrlarga yoyilmasidan $z=\pi x$ bo’lganda hosil qilinadi. Demak quyidagi

$$B(x,1-x)=\int_0^{+\infty}\frac{u^{x-1}}{(1+u)}du=\frac{\pi}{\sin\pi x},\quad(0<x<1)\quad(10)$$

Formula o’rinli. Xususiy holda, agar x=1-x=$\frac{1}{2}$ deb olinsa, u holda

$$B\left(\frac{1}{2},\frac{1}{2}\right)=\pi\quad(25')$$

Qiymatini olamiz.[1-15]

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