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"MATHEMATICAL MODEL FOR SEMANTIC ANALYSIS OF EXERCISES"

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Annotation. *The rapid development of digital educational technologies has created new demands for the automation and intelligent analysis of educational content. Across the world, and especially in countries undergoing rapid digitalization such as Uzbekistan, educational institutions are increasingly deploying e-learning platforms, digital textbook repositories, and automated assessment systems. These platforms generate and manage enormous volumes of exercises, questions, and instructional tasks that must be organized, classified, and evaluated in a principled manner. Without a rigorous approach to understanding the semantic content of exercises, such platforms cannot effectively adapt to the needs of individual learners, align with curriculum standards, or provide meaningful feedback to educators and policymakers. This article proposes a mathematical model for the semantic analysis of exercises used in automated learning systems. The model is built on three complementary components: vector-space representation of textual content, probabilistic dependency graphs that capture the grammatical and relational structure of exercise statements, and ontological mapping of domain-specific concepts drawn from official curriculum taxonomies. Together, these components allow the model to classify exercises by cognitive complexity, assess their semantic relevance to defined learning objectives, and evaluate their internal structural coherence. Experimental results obtained from a corpus of over four thousand Uzbek-language educational exercises demonstrate the effectiveness of the approach, yielding an average classification accuracy of 91.4% across STEM subject domains.*

The study makes a foundational contribution to the theoretical basis of intelligent tutoring systems and adaptive learning platforms in Uzbekistan.

Keywords: *semantic analysis, mathematical model, exercise classification, vector space model, ontological mapping, adaptive learning, intelligent tutoring systems, digital education, Uzbekistan.*

Аннотация. *Стремительное развитие цифровых образовательных технологий сформировало новые требования к автоматизации и интеллектуальному анализу образовательного контента. Во всём мире, и особенно в странах, активно проходящих процессы цифровизации, таких как Узбекистан, образовательные учреждения всё шире внедряют платформы электронного обучения, цифровые репозитории учебников и системы автоматизированного оценивания. Эти платформы генерируют и обрабатывают огромные объёмы упражнений, вопросов и учебных заданий, которые необходимо систематизировать, классифицировать и оценивать на научно обоснованной основе. Без строгого подхода к пониманию семантического содержания заданий такие системы не способны эффективно адаптироваться к потребностям отдельных обучающихся, обеспечивать соответствие образовательным стандартам и предоставлять содержательную обратную связь преподавателям и разработчикам образовательной политики.*

В данной статье предлагается математическая модель семантического анализа упражнений, используемых в автоматизированных обучающих системах. Модель основана на трёх взаимодополняющих компонентах: векторном представлении текстового содержания, вероятностных графах зависимостей, отражающих грамматическую и логико-смысловую структуру формулировок заданий, а также онтологическом отображении предметно-специфических понятий, сформированном на основе официальных образовательных таксономий. Совокупность этих компонентов позволяет классифицировать упражнения по уровню когнитивной сложности, оценивать их семантическое соответствие заданным образовательным целям и анализировать внутреннюю структурную согласованность.Экспериментальные результаты, полученные на корпусе из более чем четырёх тысяч учебных упражнений на узбекском языке, подтверждают эффективность предложенного подхода: средняя точность классификации в дисциплинах STEM составила 91,4 %. Исследование вносит фундаментальный вклад в развитие теоретических основ интеллектуальных обучающих систем и адаптивных образовательных платформ в Узбекистане.

Ключевые слова: *семантический анализ, математическая модель, классификация упражнений, векторная модель пространства, онтологическое отображение, адаптивное обучение, интеллектуальные обучающие системы, цифровое образование, Узбекистан.*

The emergence of intelligent educational platforms has transformed the way instructional content is created, organized, and delivered. Traditional approaches to exercise design relied on the expertise and intuition of individual teachers who selected and sequenced tasks based on their professional experience. While this approach has served education well for centuries, it does not scale to the demands of contemporary digital learning environments, where a single platform may serve hundreds of thousands of learners simultaneously across diverse subjects, age groups, and proficiency levels. The need for automated tools capable of understanding the meaning and purpose of educational exercises has therefore become one of the central challenges in the field of educational technology [1, 12-p]. The concept of semantic analysis refers to the computational extraction and representation of meaning from natural language text. In the context of educational exercises, semantic analysis encompasses a range of tasks: determining what subject matter an exercise addresses, identifying the cognitive operation it demands of the learner, assessing the clarity and coherence of its formulation, and establishing its relationship to broader learning objectives defined by a curriculum. These tasks are far more nuanced than simple keyword matching or topic labeling; they require a model that can capture the contextual, relational, and structural dimensions of meaning in exercise text. Existing approaches to semantic analysis in educational settings have largely been developed for the English language and for curricula prevalent in North America and Western Europe. These approaches are not readily applicable to the educational context of Uzbekistan for two principal reasons. First, the Uzbek language is an agglutinative Turkic language with rich morphological structure, where a single root word may generate dozens of inflected forms that carry distinct grammatical and semantic meanings [2, 45-p].

Standard tokenization and representation techniques designed for English or other analytic languages do not perform well on Uzbek text without substantial adaptation. Second, the curriculum structure and pedagogical traditions of Uzbekistan reflect a distinct national educational heritage that requires its own formal ontological representation, rather than a simple translation of foreign curriculum taxonomies.The national digitalization agenda of the Republic of Uzbekistan, as articulated in the Presidential Decree PD-60 of 2022 on the development of digital education and the Strategy "Digital Uzbekistan 2030," has made the development of intelligent educational platforms a state priority. The Ministry of Public Education has initiated the digitization of all secondary school textbooks and the deployment of a national e-learning portal.

However, the intelligent features of these platforms — adaptive content delivery, personalized exercise recommendation, automated difficulty calibration — require precisely the kind of semantic analysis infrastructure that this article aims to provide. Without a formal semantic model grounded in the Uzbek language and the national curriculum, these intelligent features cannot be realized in a reliable and pedagogically valid way [3,78-p]. This article addresses the gap by presenting a comprehensive formal model that integrates multiple complementary methods of semantic representation. The model is designed to be mathematically principled, computationally feasible without requiring the massive computational resources demanded by large neural language models, and adaptable to the Uzbek-language educational context.

It has been developed through close collaboration with subject matter experts from the faculties of mathematics and informatics at the National University of Uzbekistan, and has been evaluated on a corpus of exercises drawn from official Uzbekistan secondary school textbooks.

The scientific novelty of the work lies in the integration of three previously separate approaches — distributional semantics, dependency-based structural analysis, and ontological concept mapping — into a unified semantic model tailored for educational exercise analysis in the Uzbek-language setting. Previous work in Uzbekistan on natural language processing and educational data mining has addressed these components separately, but no prior study has combined them into a single coherent framework oriented toward exercise semantics [4,33-p]. The present article fills this gap and provides a reusable theoretical foundation for future work in educational artificial intelligence in Uzbekistan.

The remainder of the article is organized as follows. Section 2 provides a review of related work in semantic analysis, educational data mining, and intelligent tutoring systems.

Section 3 introduces the formal components of the proposed model, explaining the conceptual basis and practical rationale for each component. Section 4 describes the computational methods used to operationalize the model. Section 5 presents the experimental evaluation on Uzbek-language educational corpora. Section 6 discusses the results and their implications for educational practice and future research. Section 7 concludes the article.

The automatic analysis of educational exercises and questions has a history stretching back to the early days of computer-assisted instruction in the 1960s and 1970s.

Early systems used simple pattern-matching rules to determine whether a student-provided answer matched a set of expected response templates.

While effective for highly constrained domains such as arithmetic drill, these rule-based systems could not generalize to open-ended questions or exercises formulated in natural language.

The fundamental limitation was the assumption that the meaning of an exercise could be captured by its surface form — the specific words and grammatical structures used — rather than by the underlying semantic content it conveyed [5,19-p].Subsequent work in the 1980s and 1990s drew on advances in computational linguistics to develop more sophisticated approaches to exercise understanding. Syntactic parsers capable of analyzing the grammatical structure of English sentences were applied to exercise texts to extract predicate-argument structures.

These structures — representing who does what to whom — provided a richer basis for semantic comparison than raw text matching. However, the parsers of this era were brittle and error-prone, and the resulting systems required extensive hand-crafted lexicons and grammar rules that were expensive to develop and maintain. A significant advance came with the introduction of the Vector Space Model (VSM) and its application to educational content analysis. The core insight of the VSM is that the meaning of a text can be approximated by the statistical distribution of the words it contains across a large corpus.

Texts that use similar words in similar contexts are likely to be about similar topics and to require similar cognitive operations from the learner. This distributional hypothesis, first articulated by linguists in the 1950s, was operationalized computationally through term-frequency weighting schemes and dimensionality reduction techniques [6,55-p].

Latent Semantic Analysis, introduced in the early 1990s, represented a major refinement of the basic VSM approach. By applying Singular Value Decomposition to the term-document matrix, LSA identified latent semantic dimensions underlying the surface vocabulary of a corpus, allowing it to capture synonymy and polysemy to a degree that raw term-frequency representations could not. LSA was successfully applied to educational tasks including automated essay scoring, exercise coherence evaluation, and reading comprehension difficulty assessment.

These early successes demonstrated the potential of distributional semantic methods for educational content analysis, though the lack of structural and relational information in LSA representations remained a significant limitation.

Ontology-based approaches to educational content analysis were developed in response to the recognized limitations of purely distributional methods. An ontology is a formal, structured representation of the concepts and relations in a domain of knowledge.

In the context of education, ontologies have been used to represent curriculum structures, learning objectives, competency frameworks, and the prerequisite relationships among concepts that determine the logical sequencing of instruction [7,102-p].Research in the field of intelligent tutoring systems demonstrated that ontological representations could substantially improve the accuracy of exercise classification and the reliability of adaptive content recommendation. By mapping exercises to concepts in a curriculum ontology, it became possible to determine not only what subject matter an exercise addressed, but also what prerequisite knowledge a student would need to approach it successfully and what further concepts it would prepare them to learn. This capability is essential for the design of truly adaptive learning systems that can sequence instruction optimally for each individual learner.

In Uzbekistan, early work on ontological modeling of educational content was conducted at Tashkent State Technical University and the National University of Uzbekistan. These studies focused primarily on ontologies for the domains of informatics and engineering, and demonstrated the feasibility of constructing structured knowledge representations aligned with the Uzbekistan national curriculum [8,74-p].

However, these ontologies were developed in isolation from semantic text analysis methods, and their integration with computational linguistics tools for Uzbek text processing remained an open problem.The past decade has seen the rise of deep neural network approaches to natural language processing, which have achieved state-of-the-art performance on a wide range of semantic analysis tasks. Pre-trained language models based on the transformer architecture — such as BERT, RoBERTa, and their multilingual variants — have shown strong performance on tasks including semantic similarity assessment, question difficulty prediction, and topic labeling in educational content [9,28-p].However, these neural models present significant challenges for deployment in the Uzbekistan educational context. They require enormous computational resources for training and inference, making them impractical for deployment on the modest server infrastructure available in many Uzbek schools and regional education centers. They also require large annotated training corpora, which do not yet exist for Uzbek-language educational content.

Furthermore, neural models are largely opaque in their reasoning, providing predictions without interpretable explanations — a serious drawback in educational settings where teachers and curriculum designers need to understand and trust the basis of the system's recommendations.

The model proposed in this article is designed to occupy the middle ground between the oversimplified keyword-matching approaches of early educational AI and the computationally demanding neural approaches of the current era. It combines the mathematical rigor and interpretability of classical distributional semantic and ontological methods with modern insights from dependency-based structural analysis, yielding a system that is both theoretically principled and practically deployable in the Uzbek educational context.Before presenting the components of the proposed model, it is necessary to clarify what we mean by the semantics of an educational exercise. An exercise is not merely a piece of text; it is a communicative act that encodes a pedagogical intention.

The author of an exercise intends for it to elicit a particular type of cognitive engagement from the learner — to remember a definition, to apply a procedure, to analyze a situation, or to construct an argument. The semantic content of the exercise includes both the propositional content — the subject matter it concerns — and the illocutionary force — the type of cognitive act it demands [10,47-p]. This dual character of exercise semantics has important implications for the design of an analysis model. A model that captures only propositional content — what the exercise is about — cannot distinguish between an exercise that asks a student to recall a formula and one that asks them to derive it from first principles. Yet this distinction is crucial for curriculum alignment, adaptive sequencing, and cognitive difficulty assessment. The proposed model therefore incorporates both a representation of topical content and a representation of the cognitive operation demanded by the exercise, integrating them into a unified formal framework.

A further dimension of exercise semantics concerns the internal coherence and clarity of the exercise statement.

An exercise that is semantically incoherent — that combines unrelated concepts without a clear logical thread, or that uses terminology inconsistently — will confuse learners and produce unreliable assessment data. The model therefore includes a coherence component that evaluates the internal semantic consistency of an exercise statement, enabling the identification of poorly formulated exercises before they are deployed in assessment systems.

The proposed model uses Bloom's Revised Taxonomy of Educational Objectives as the primary framework for classifying exercises by cognitive complexity. Bloom's Taxonomy, originally developed in 1956 and revised by Anderson and Krathwohl in 2001, organizes cognitive learning objectives into six hierarchical levels: Remember, Understand, Apply, Analyze, Evaluate, and Create. Each level represents a qualitatively distinct mode of cognitive engagement with subject matter, with the higher levels requiring more sophisticated integration of knowledge and reasoning [11,63-p].The utility of Bloom's Taxonomy as a framework for exercise classification lies in its widespread adoption in curriculum design and its well-developed operationalization through action verbs. Exercises at the Remember level are typically formulated using verbs such as "list," "name," "define," or "recall." Exercises at the Apply level use verbs such as "calculate," "solve," "use," or "demonstrate." Exercises at the Create level use verbs such as "design," "construct," "formulate," or "develop." These verb patterns provide an important linguistic signal for the cognitive level of an exercise, which the model exploits through its analysis of the dependency structure of the exercise statement.

In the Uzbek-language educational context, Bloom's Taxonomy has been adopted by the Ministry of Public Education as the official framework for learning objective formulation in the national curriculum. This adoption means that the model's use of Bloom's levels as a classification target is directly aligned with the operational needs of Uzbek curriculum designers and educational platform developers [12,88-p]. The alignment between the model's theoretical framework and the practical vocabulary of Uzbek educational policy is a significant advantage for the model's adoption and deployment. A domain ontology is a formal, machine-readable representation of the concepts, properties, and relations that constitute a field of knowledge. In the context of school-level education, a domain ontology encodes the concepts that appear in the curriculum — mathematical operations, physical phenomena, programming constructs, historical events — along with the semantic relations between them, such as generalization hierarchies, prerequisite relationships, and application contexts [13,91-p].The domain ontology plays two essential roles in the proposed model. First, it serves as the reference structure for mapping the vocabulary of an exercise to formal curriculum concepts. When the model encounters the word "integral" in a mathematics exercise, the ontology specifies that this term refers to the concept of Riemann integration, which belongs to the domain of mathematical analysis, is a prerequisite for the study of differential equations, and is associated with learning objectives at the Apply and Analyze levels of Bloom's Taxonomy. This rich contextual information, encoded in the ontology, allows the model to go far beyond simple keyword identification. Second, the ontology provides the framework for computing the alignment between an exercise and a learning objective.

Learning objectives in the Uzbekistan national curriculum are defined in terms of the concepts and competencies that students are expected to acquire. By mapping both exercises and learning objectives to the same ontological concept space, the model can compute a principled alignment score that reflects the degree to which an exercise addresses the competencies specified by a given objective.

This capability is fundamental for the automated audit of exercise repositories and for the intelligent sequencing of instructional content in adaptive learning platforms.

The first component of the model is responsible for transforming the raw text of an exercise into a numerical representation that can be processed by computational methods.

This transformation begins with a preprocessing pipeline that applies a series of linguistic normalizations to the exercise text. The pipeline includes sentence segmentation, word tokenization, morphological normalization, stopword removal, and term weighting [14, 5-p]. For Uzbek-language exercises, the morphological normalization step is particularly critical. Uzbek is an agglutinative language in which grammatical information — tense, aspect, case, number, possession, and so forth — is encoded through a series of suffixes attached to the root of a word.

A single verb root may give rise to dozens of distinct inflected forms in different grammatical contexts. Without morphological normalization, each of these forms would be treated as a different term by the model, vastly inflating the vocabulary size and reducing the ability to identify semantic relationships between exercises that use the same root words in different grammatical forms.The morphological analyzer used in the model was developed at the Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan and has been validated on a corpus of over two million Uzbek words [15, 22-p]. It operates by segmenting each inflected token into its root and a sequence of morphological suffixes, then returning the canonical root form for use in downstream processing. This normalization ensures that the model can recognize the semantic relationship between, for example, the exercise formulations "hisobla" (calculate) and "hisoblanadi" (is calculated), which share the root "hisob" but differ in voice and mood.

After preprocessing, the normalized tokens of each exercise are used to construct a vector representation in a high-dimensional term space. The weighting scheme assigns greater importance to terms that are frequent within the exercise but rare across the exercise corpus as a whole, reflecting the intuition that terms that distinguish this exercise from others in the corpus are more semantically informative than ubiquitous terms that appear in nearly every exercise. The resulting weighted term vectors form the basis for computing semantic similarity between exercises and between exercises and learning objective descriptions. The second component of the model analyzes the grammatical structure of the exercise statement to extract information about the cognitive operation it demands. This analysis is based on the syntactic dependency structure of the exercise, which represents the grammatical relationships between the words of the statement in the form of a directed graph [16, 38-p].

In a dependency graph, each word of the sentence is represented as a node, and each grammatical relation between two words is represented as a directed arc from the head word to the dependent word, labeled with the type of grammatical relation it expresses.

The root of the dependency tree is typically the main verb of the sentence, which represents the primary predicate — the action or state that the exercise is asserting or requesting.

For educational exercises, the main verb carries particularly rich semantic information, as it directly encodes the type of cognitive operation the exercise requires.The model extracts the main predicate verb of each exercise and classifies it according to a taxonomy of cognitive operation types derived from the action verb lists associated with Bloom's Taxonomy levels. Verbs such as "define," "list," and "name" are classified as recall operations; verbs such as "explain," "describe," and "summarize" are classified as comprehension operations; verbs such as "calculate," "apply," and "solve" are classified as application operations; and so forth. This classification is performed using a lexicon of Uzbek-language action verbs annotated by educational experts at the National University of Uzbekistan [17, 56-p].Beyond the main predicate, the dependency analysis also extracts the principal arguments of the predicate — the subject, object, and complement of the main verb — and the modifiers that constrain these arguments. Together, these structural features constitute what the model terms the operational signature of the exercise: a compact representation of the type of action demanded and the conceptual entities it is directed toward. The operational signature provides the model with a structural perspective on exercise semantics that complements the topical perspective provided by the textual representation component. The third component of the model maps the content of each exercise to a set of formal concepts drawn from the domain ontology. This ontological mapping transforms the exercise from a representation in terms of surface vocabulary to a representation in terms of the structured conceptual vocabulary of the curriculum, enabling principled comparisons between exercises and between exercises and learning objectives [18, 67-p].The mapping process begins by identifying which terms in the normalized token sequence of the exercise correspond to concepts in the domain ontology. This lookup is performed against an index that maps terms and their synonyms to ontology concept identifiers. Because the Uzbek educational vocabulary includes both native Uzbek terms and borrowed terms from Russian and Arabic, the ontology index includes entries for all common variants of each concept's name.When a term in the exercise corresponds to multiple candidate concepts in the ontology — a situation that arises frequently due to polysemy and terminological ambiguity — the model uses the broader context of the exercise to disambiguate. Specifically, the surrounding terms and the subject domain label of the exercise (mathematics, physics, informatics, etc.) are used to select the most contextually appropriate concept from among the candidates. This context-sensitive disambiguation substantially improves the accuracy of the ontological mapping compared to naive lexical lookup [19, 44-p].

The result of the ontological mapping is a set of curriculum concepts associated with each exercise. These concept sets are used in two ways. First, they provide additional features for the cognitive complexity classification task, capturing the subject-domain information that complements the cognitive operation information provided by the dependency analysis. Second, they serve as the primary basis for computing the alignment between exercises and learning objectives, since both exercises and objectives are ultimately expressed in terms of the same set of ontological concepts. To evaluate the proposed model, we constructed a corpus of educational exercises drawn from official sources in the Uzbekistan educational system.

The corpus includes exercises from three subject domains: mathematics, informatics, and physics, sourced from the digital textbook repository of the Ministry of Public Education of the Republic of Uzbekistan and from published secondary school textbooks by Uzbek authors [20, 13-p]. The mathematics exercises were drawn from textbooks authored by Mirzaahmedov, the informatics exercises from textbooks authored by Shamsiyev and Yuldashev, and the physics exercises from textbooks authored by Qodirov.The corpus was manually annotated by a team of six subject matter experts recruited from the faculties of mathematics, informatics, and physics at the National University of Uzbekistan. Each exercise was independently annotated by two experts for three attributes: its Bloom's Taxonomy level, its primary learning objective alignment according to the national curriculum, and its semantic coherence on a five-point scale ranging from "highly incoherent" to "fully coherent." Disagreements between the two annotators were resolved through a third-party adjudication procedure involving a senior faculty member from the relevant discipline. The resulting corpus contains a total of 4,200 annotated exercises, of which 1,800 are mathematics exercises, 1,400 are informatics exercises, and 1,000 are physics exercises.

The distribution across Bloom's Taxonomy levels is not uniform, reflecting the actual distribution in the textbooks: exercises at the Remember and Apply levels are most common, accounting for approximately 65% of the corpus, while exercises at the Evaluate and Create levels are relatively rare, accounting for approximately 8%. This distribution poses a challenge for classifier training but accurately reflects the real-world distribution that any deployed system would encounter [21, 81-p].The model was implemented in Python using the NLTK and scikit-learn libraries for text processing and machine learning, and a custom Uzbek-language morphological analysis module built on the finite-state transducer developed at the Institute of Mathematics of the Academy of Sciences of Uzbekistan [22,7-p]. The domain ontology was implemented using the OWL Web Ontology Language and accessed via the owlready2 library.

The dependency parser was a transition-based neural parser trained on the Uzbek Universal Dependencies treebank, achieving a Labeled Attachment Score of 84.2% on the held-out test set.The textual representation was computed using a vocabulary of the 5,000 most informative terms in the corpus, selected by term frequency-inverse document frequency weighting after morphological normalization. The ontology used in the experiments contains 2,850 concept nodes and 6,140 relational edges organized into three subject domain hierarchies corresponding to the three disciplines represented in the corpus. The cognitive operation lexicon contains 312 annotated Uzbek-language action verbs organized into six Bloom level categories. The exercise classification model was evaluated using stratified ten-fold cross-validation, with results reported as macro-averaged F1 scores across all six Bloom Taxonomy levels. The baseline model using only the textual representation component achieved a macro F1 score of 74.3%, demonstrating that distributional semantic features alone provide a reasonable but incomplete basis for cognitive complexity classification. Adding the dependency-based operational signature features improved the macro F1 score to 82.6%, a substantial gain that reflects the importance of structural analysis for capturing the cognitive dimension of exercise semantics [23, 3-p]. The full model incorporating all three components — textual representation, dependency analysis, and ontological concept mapping — achieved a macro F1 score of 91.4%, representing a statistically significant improvement over both baseline configurations. Performance varied across Bloom Taxonomy levels, with the Remember and Create categories achieving the highest F1 scores of 95.1% and 93.8% respectively. The high performance on Remember-level exercises reflects the strong lexical signals provided by recall-oriented action verbs in Uzbek, which are highly distinctive and reliably identified by both the dependency analysis and the textual representation. The high performance on Create-level exercises reflects the fact that such exercises, though rare, tend to use a distinctive set of high-level action verbs and complex conceptual vocabulary that distinguishes them clearly from lower-level exercises. The Analyze level proved the most challenging for classification, achieving an F1 score of 86.2%. Qualitative analysis of the misclassified exercises revealed that the primary source of confusion was between the Analyze and Evaluate levels, which share many surface features in their Uzbek-language formulations and differ primarily in the degree of metacognitive reflection they demand — a distinction that is difficult to capture without deeper pragmatic analysis of the exercise context.The learning objective alignment task achieved a mean average precision of 0.883, indicating strong retrieval performance. The coherence scoring component showed a Pearson correlation of 0.79 with human-assigned coherence ratings, validating its utility as a practical quality assessment tool for exercise authoring workflows.

The experimental results provide strong evidence for the effectiveness of the proposed semantic analysis model. The most important finding is that the combination of three complementary semantic representations — distributional, structural, and ontological — substantially outperforms any single representation in isolation. This result confirms the theoretical motivation for the integrated model design: the semantics of educational exercises is a multi-dimensional phenomenon that cannot be fully captured by any single analytical lens. The substantial gain achieved by adding the ontological concept features — from 82.6% to 91.4% macro F1 — is particularly noteworthy. It suggests that formal curriculum knowledge, encoded in the ontology, provides information that cannot be inferred from the exercise text alone. This finding has an important practical implication: the investment required to construct and maintain domain ontologies aligned with the national curriculum is justified by the significant gains in classification accuracy that such ontologies enable.We recommend that future national educational AI initiatives in Uzbekistan prioritize the development of comprehensive, maintained curriculum ontologies as a shared public infrastructure.The proposed model has several direct practical implications for educational content management and delivery in Uzbekistan. First, the cognitive complexity classification capability enables automated auditing of exercise repositories to ensure that the distribution of Bloom Taxonomy levels in a given course or module aligns with the intended pedagogical design. Research in educational psychology has consistently shown that effective instruction requires a carefully calibrated balance of exercises at different cognitive levels, with lower-level exercises providing the foundational knowledge base needed for higher-level tasks. Second, the learning objective alignment capability provides a principled basis for automated exercise recommendation in adaptive learning platforms. When a student has demonstrated mastery of some learning objectives and is working toward others, the model can identify exercises that are precisely aligned with the target objectives and at the appropriate cognitive level — a capability that is fundamental to truly personalized instruction.

This use case is directly relevant to the national e-learning platform being developed under the Digital Uzbekistan 2030 initiative. Third, the coherence scoring capability offers practical value for exercise quality assurance workflows. Textbook authors and curriculum designers can use the coherence score to identify exercises that may be ambiguous or internally inconsistent before publication, reducing the risk of deploying flawed assessment items that produce unreliable data about student learning. Several limitations of the present study should be acknowledged.

The domain ontology constructed for this research covers three subject areas of the secondary school curriculum. Extending the model to the full range of subjects in the Uzbekistan national curriculum — including history, literature, geography, biology, chemistry, and foreign languages — will require substantial additional ontology engineering effort. This extension is planned as part of a multi-year research program funded by the Ministry of Innovative Development. The dependency parser used in the experiments, while state-of-the-art for Uzbek, achieves a Labeled Attachment Score of 84.2%, which means that approximately 16% of grammatical relations in the exercise texts are assigned incorrect dependency labels. Errors in dependency parsing propagate through the model and contribute to misclassifications, particularly at the Analyze and Evaluate levels where the fine-grained structural analysis of the exercise predicate is most important. Future work will address this limitation by developing an exercise-domain-adapted parser trained on a treebank of annotated educational texts. The model also does not currently handle exercises that incorporate visual elements — diagrams, graphs, geometric figures, or data tables — which are common in mathematics and physics exercises. The semantic analysis of such multimodal exercises requires image understanding capabilities that are beyond the scope of the present study. Initial work on integrating visual content analysis into the framework has been undertaken and will be reported in a forthcoming publication.

This article has presented a formal model for the semantic analysis of educational exercises, developed in the context of the Uzbekistan national education system and evaluated on a corpus of Uzbek-language exercises from secondary school mathematics, informatics, and physics. The model integrates three complementary components — vector-space textual representation, dependency-based structural analysis, and ontological concept mapping — into a unified framework that supports cognitive complexity classification, learning objective alignment, and semantic coherence scoring. Evaluation results demonstrated that the integrated model achieves a macro-averaged F1 score of 91.4% for cognitive complexity classification, substantially outperforming models that use any single component in isolation. The theoretical contribution of the work lies in the formalization of exercise semantics as a multi-dimensional construct that encompasses topical content, cognitive operation type, and internal structural coherence, and in the design of an integrated computational model that captures all three dimensions simultaneously. The practical contribution lies in the provision of a deployable, interpretable, and computationally efficient tool for exercise analysis that is adapted to the linguistic and curricular characteristics of the Uzbek educational context.The model is designed to serve as a foundational component of intelligent educational platforms in Uzbekistan, directly supporting the national digitalization agenda embodied in Presidential Decree PD-60 and the Digital Uzbekistan 2030 strategy.

Future work will extend the model to additional subject domains, improve the underlying Uzbek-language processing tools, and integrate multimodal content analysis. The annotated corpus and domain ontology developed for this research will be made publicly available to support further research in Uzbek-language educational artificial intelligence.

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