# EXAMINING POWER SERIES IN COMPLEX NUMBERS

**Abdul Wares Sediqy**
Physics Department, Education faculty, Farah University, Farah, Afghanistan.
*Email address:* ***ab.sediqy812@gmail.com***
**Jawid Hashemi**
Mathematics Department, Education faculty, Farah University, Farah, Afghanistan.
*Email address:* ***Jawidhashemi4@gmail.com***
***https://doi.org/10.5281/zenodo.13764113***

***Abstract.*** *The subject of series is one of the most important works that has always been of interest in the field of mathematics analysis and plays a fundamental role in solving complex mathematical problems. Especially the power series of complex numbers, which has significant advantages in mathematics. The purpose of this research is to investigate and understand series of complex numbers. In this research, the latest articles, theses, treatises and numerical analysis books have been used. In the numerical analysis of series of complex numbers, it is one of the most important and widely used branches. Since no comprehensive research has been done on this issue until now and most researchers have used less series of complex numbers in their research, which series of power solves many complex problems of mathematics easily, including its applications in the root of numbers. Negative are arithmetic and geometric series, Taqat and Taylor series. In this article, open and closed library pages and authentic internet articles have been used. It is worth mentioning that this article is important for all those interested in mathematics and especially for those interested in mathematics, engineering, physics and computer science, and they can use the contents of this research in their scientific and academic life. Since series in complex numbers has different branches, I have tried to include the topics of series of complex numbers in this research so that it will be a good extension for the researchers of mathematical science in the future.*

***Key words:*** *complex numbers, circle of convergence, series, divergent, convergent.*

# ИЗУЧЕНИЕ СТЕПЕННЫХ РЯДОВ В КОМПЛЕКСНЫХ ЧИСЛАХ

***Аннотация.*** *Тема рядов является одной из важнейших работ, которая всегда представляла интерес в области математического анализа и играет фундаментальную роль в решении сложных математических задач. Особенно степенные ряды комплексных чисел, которые имеют значительные преимущества в математике. Целью данного исследования является исследование и понимание рядов комплексных чисел. В этом исследовании были использованы последние статьи, диссертации, трактаты и книги по численному анализу. В численном анализе рядов комплексных чисел это одно из самых важных и широко используемых направлений. Поскольку до сих пор не было проведено всесторонних исследований по этому вопросу, и большинство исследователей использовали меньше рядов комплексных чисел в своих исследованиях, какой ряд степеней легко решает многие сложные задачи математики, включая его приложения в корне чисел. Отрицательными являются арифметические и геометрические ряды, ряды Таката и Тейлора. В этой статье были использованы открытые и закрытые библиотечные страницы и подлинные интернет-статьи. Стоит отметить, что эта статья важна для всех, кто интересуется математикой, и особенно для тех, кто интересуется* *математикой, инженерией, физикой и информатикой, и они могут использовать содержание этого исследования в своей научной и академической жизни. Поскольку ряды комплексных чисел имеют разные ответвления, я постарался включить темы рядов комплексных чисел в это исследование, чтобы оно стало хорошим продолжением для исследователей математической науки в будущем.*

***Ключевые слова:*** *комплексные числа, круг сходимости, ряд, расходящийся, сходящийся.*

INTRODUCTION

From the beginning of human life until now, mathematics has been used as an urgent and unbreakable necessity of human life and parallel to language. According to philosophers, mathematics is the key to all sciences. In the oldest words that have been recorded in history, thinkers and scientists have advanced the science of mathematics to the peak due to their tireless work, and each of them has written a golden line in the history of mathematics.

Mathematics in general studies the pattern of structure, evolution of space. In fact, if we want to speak in a simpler language, we should say that it is the study of shapes and numbers. In mathematics, instead of using words and punctuation marks, numbers and symbols are usually used.

Numerical analysis is a part of the science of mathematics that deals with setting, checking and applying approximate methods of calculation to solve those problems of continuous mathematics (as opposed to discrete mathematics) that cannot be solved by analytical and exact methods. Some problems of numerical calculations come directly from calculus. A number of continuous mathematics problems can be solved exactly by an algorithm, which are known as direct problem solving methods. For example, we can mention Gaussian elimination method for solving the system of linear equations as well as the non-composite algorithm used in linear programming. On the contrary, there is no direct solution method for many problems and other methods such as the iterative method should be used.

Brooke Taylor was one of the mathematicians whose career gave a special face to numerical analysis. Taylor's series, which generally presents functions in the form of power series, is very important in approximate calculations, such as numerical calculations, integrals, square roots, obtaining non-rational numbers, solving ambiguity in limits, solving differential equations, etc.

Illusory numbers were invented by mathematicians in the 16th century to determine the general solution of quadratic and cubic equations. Since the square of any real number is equal to zero or a positive number, as a result, the equation $x^2 = -1$ cannot be solved in the field of real addition. In 1777, Leonard Dower introduced the symbol i with the characteristic $i^2 = -1$ . As a result, the solutions of the above equation are equal to ±i and the symbol i is called imaginary.

Series and rows of imaginary numbers have many similarities with series and rows of real numbers. In this article, we try to introduce the concepts of convergence and divergence of strong series of complex numbers. Many power series that can be explained in the field of real numbers can also be explained in the field of complex numbers. Even many relationships can be explained in a simpler way with their help. From this sentence, we can mention the radius of convergence of the circle of convergence, which can be explained in a clearer way in the power series.

**The importance of research**

The necessity of each subject depends on its application, and this research can be expressed in relation to the investigation of the power series in complex numbers, and researchers should gain familiarity in the mentioned fields, so our effort has been to cover the corners of this subject in this article. Let's take a look.

**Series**

Suppose the synonyms of an infinite number of numbers $u_1, u_2, u_3 \dots u_n$ are given:

$$\sum_{n=1}^{\infty} u_n = u_1 + u_2 + u_3 \dots u_n \dots$$

is called a numerical series, and the numbers $u_1$, $u_2$, $u_3$... $u_n$are called the limits of the series.

**Definition:**

The sum of the n values of the first term n=1,2,3... of the above series is given to S_n and displayed as $S_n = u_1 + u_2 + u_3 \dots u_n$

**Convergence of series:**

If the limit $S_n$ approaches n $\to \infty$, a number is determined, that is:

$$\lim_{n\to\infty} S_n = S$$

In this case, the mentioned series is convergent and the number S is the sum of the given series (Farahi, 2018).

**Power series**

Power series according to power x is a function series that has the following form:

$$\boldsymbol{a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots a_nx^n}$$

**Note:** Every power series is convergent for S=0.

Convergence radius of strength series

If $\sum_{n=1}^{\infty} u_n x^n$, its radius of convergence is equal to:

$$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| \qquad \text{(Khair Elahi, 2015)}$$

**Complex functions**

A symbol such as Z that is substituted for each member of a set of complex numbers such as S is called a complex variable. If we assign to each variable Z in S one or more values of a complex variable such as w, then the set of ordered pairs:

f={(w,z):zϵs}

is called a complex function on S and is represented by w=f(z). The variable z is called the independent variable and w is called the dependent variable (Stewart, 1914).

Power series of complex numbers

Many power series can also be explained in the field of real numbers in the field of complex numbers, even many relationships can be explained in a better and simpler way with the help of them, among which we can mention the radius of convergence, the concept mentioned in the power series can be seen and explained more clearly (Talabaz, 2013).

ISSN: 2181-3906 2024
International scientific journal
«MODERN SCIENCE AND RESEARCH»
VOLUME 3 / ISSUE 9 / UIF:8.2 / MODERNSCIENCE.UZ

Definition:

A series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ while $a_n \subseteq ¢$ is a synonym of complex numbers $z \wedge z_0 \in ¢$ is given, called the strength series with expansion in $z_0$ (Dost, 2015).

theory

For the series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ if the series converges at a point $z_1 \neq z_0$, then for all $|z-z_0| \leq |z_1 - z|$ Absolute is convergent (Silverman, 1996).

If the series is divergent in one $z_2$ , then for all z's it becomes $|z_2 - z_0| \leq |z - z_0|$It is distant.

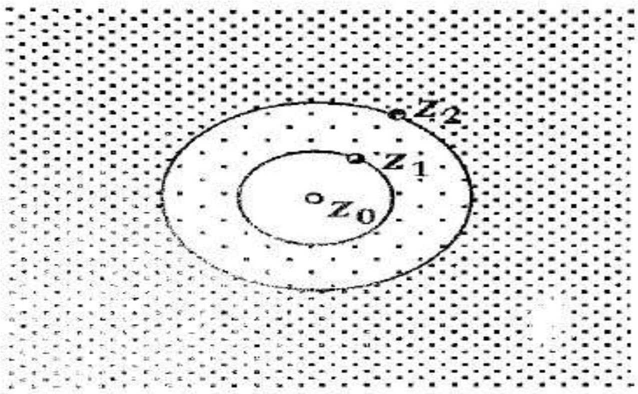


Figure (1) radius of convergence in complex numbers (Talabaz, 2011).

Proof: whenever $z_1 \neq z_0$ is a point of convergence and z with $|z-z_0| \leq |z_1 - z|$ is given, in this case, from the approximation $\sum_{n=1}^{\infty} a_n(z-z_0)^n$, we find that $\lim_{n\to\infty}(z_1-z_0)^n = 0$ and there exists an $M > 0$ such that $|a_n(z_1-z_0)^n| \leq M$ , $\sqrt{n} \in N \cup \{0\}$ also because:

$$|a_n(z_1-z_0)^n| = |a_n(z_1-z_0)^n| \left|\frac{z-z_0}{z_1-z_0}\right| \leq M \left|\frac{z-z_0}{z_1-z_0}\right|^n$$

and considering that $\left|\frac{z-z_0}{(z_1-z_0}\right| < 1$, therefore, the theory of the comparative condition of the series, the absolute convergence of the series is obtained from it ( Ghori, 2018).

Suppose $z_0$ is a divergence point and a z with $|z-z_0| > |z_2 - z_0|$ is given, in this case, if the series is convergent in z, then according to the theorem, $z_2$ must also be a point of convergence, and this is a contradiction with the hypothesis, which also proves the validity of the continuation of the theorem (Churchill, 2015).

**The radius of approximation of Power series**

For the power series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ , R is the radius of convergence of the region in which the series converges:

$$R = Sup\left\{|z-z_0|, \sum_{n=1}^{\infty} a_n|(z-z_0)|^n\right\}$$

Series of complex numbers, like series of real numbers, also have a circle of convergence.

The circle of convergence of the series has the center $z_0$ and is $|z-z_0| = R$ This series is convergent for every z inside this circle and divergent for z outside the circle (Zahedani, 1996).

Properties of complex Power series

- If the series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ is multiplied by a number, there is no change in its convergence and divergence.
- If the series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ is absolutely convergent, the set of its elements is equal to the first set regardless of the order we change.
- We have:

$$\sum_{n=1}^{\infty} a_n(z-z_0)^n \pm \sum_{n=1}^{\infty} b_n(z-z_0)^n = \sum_{n=1}^{\infty} (a_n \pm b_n)(z-z_0)^n$$

- If the series $\sum_{n=1}^{\infty} a_n(z-z_0)^n$ and $\sum_{n=1}^{\infty} b_n(z-z_0)^n$ have the same radius convergence, then the series $\sum_{n=1}^{\infty} (a_n \pm b_n)(z-z_0)^n$ also has the same radius of convergence.
- Two series can be multiplied or divided together.

Example (1)

For example, the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is equal to R→∞

$$\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \left|\frac{1}{n!}.\frac{(n+1)!}{1}\right| = \lim_{n\to\infty} |n+1| = \infty$$

Example (2)

The radius of convergence $\sum_{n=0}^{\infty} n!.z^n$ is equal to zero, that is, R=0

$$\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \left|\frac{n!}{(n+1)!}\right| = \lim_{n\to\infty} \left|\frac{1}{n+1}\right| = 0$$

That is, the series converges only for $z_0 = 0$

Example (3)

The series $\sum_{n=0}^{\infty} \frac{z^n}{n}$ and $\sum_{n=0}^{\infty} z^n$ each have a radius of convergence $z = 1$

**Derivation of Power series**

Power series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ can be measured limit by limit in the convergence circle $z-z_0| < R$ took the derivative, that is:

$$\frac{d}{dx}\sum_{n=0}^{\infty} a_n(z-z_0)^n = \sum_{n=0}^{\infty} a_n \frac{d}{dx}(z-z_0)^n = \sum_{n=0}^{\infty} a_n(z-z_0)^{n-1}$$

By applying the ratio test, it is shown that the main series and its derivative are two series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ and $\sum_{n=0}^{\infty} a_n(z-z_0)^{n-1}$ have the same circle of convergence | $|z-z_0| = R$, because the power series is an infinite set of differentiable functions in a given circle.

**Integration of power series**

The power series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ can be limit to limit in the convergence circle $z-z_0| < R$ took the integral, that is

$$\int_c \sum_{n=0}^{\infty} a_n(z-z_0)^n = \sum_{n=0}^{\infty} a_n \int_c (z-z_0)^n = \sum_{n=0}^{\infty} \frac{a_n}{n+1}(z-z_0)^{n+1} + c$$

while $c$ is inside the circle $|z-z_0|$ is

Again, with the help of the ratio test, it can be shown that the main series and its integral means two series

$\sum_{n=0}^{\infty} a_n (z-z_0)^n$ and $\sum_{n=0}^{\infty} \frac{a_n}{n+1} (z-z_0)^{n+1}$ have the same circle of convergence $|z - z_0| = R$.

**Conclusion**

Series in numerical analysis help a lot for the fields of physics, computer science and finance The concept of radius of convergence of series can be better understood by working with series of strength If the series $\sum_{n=1}^{\infty} a_n (z-z_0)^n$ is divergent at the point $z_2$, then for all z in while $|z_2 - z_0| \leq |z - z_0|$ is also dista Series of complex numbers also have a circle of convergence.

The radius of convergence of these series is R, which is the radius of the circle of convergence. In this case, the series is divergent for points outside the circle and convergent for points inside the circle Power series of complex numbers have properties that can be used to solve many closed problems in a simple way (Aghili, 1985).

**REFERENCES**

1. Stewart, J. (1914). Differential and integral calculators (First ed., Vol. first). Isfahan: Isfahan University Publications.
2. Churchill, R. (1315). Complex variables and their applications. Translated by Ali, A. A. (First ed). Tehran: Nogarande Danesh Publications
3. Khair Elahi, H. (2015). Solving the problems of Complex variables and its applications. Third edition. Hamedan: Student Publications.
4. Talabaz, H. (2011). Complex analysis1(first ed., vol. First). Herat: Quds Publications. First edition
5. Talabaz, H. (2014). Complex Analysis2 (First ed., vol. Second). Herat: Quds Publications.
6. Zahidani, H. Z. (1996). Complex functions (Second ed). Shiraz: Center Publications.
7. Shams, S. (1998). Complex functions and theory of analytic functions (first ed., vol. First). Urmia: Jihad Publications.
8. Aqli, A. (2005). Familiarity with Complex variables (first ed., vol. First). Guilan: Jihad Publications
9. Ghori, M. A. (2019). Complex analysis (second ed., vol. First). kabul. Saeed Publications.
10. Farahi, A. R. (2018). Complex analysis (First ed., vol. First). Kabul: Jahan Islam Publishing.
11. Silverman, R. (1996). Complex analysis and its applications. Translated by Ali O. and Khalil, F (Third ed., vol. Second). Tehran: Nile Publications
12. Dost, A. R. (2015). 4 Mathematical analysis (series). (first ed., vol. First). kabul: Nevisa Publications
13. Aghili, A. (1985). Familiarity with Complex variables (second ed., vol. First). Tehran: Tawakkol Publications
14. Talebian, S. M. (2018). Calculus of functions of several variables and vector analysis (First ed., vol. First). Tehran: Very Green Publications.